

## • Using Letters to Identify Geometric Figures





In geometry we often use letters to refer to **points.** We can identify polygons by the points at each vertex.



We may refer to this triangle as  $\triangle ABC$  ("triangle ABC"). We may also refer to this triangle in these ways:

 $\triangle$  BCA  $\triangle$  CAB  $\triangle$  ACB  $\triangle$  BAC  $\triangle$  CBA

To name a polygon, we start at one vertex and move around the perimeter, naming each vertex in order until all vertices are named. The order is important.

**Conclude** The figure below can be named rectangle *ABCD* or rectangle *ADCB* but not rectangle *ACBD*. Why not?



We name a line by naming any two points on the line. We refer to the line below as "line *AB*" (or "line *BA*").



We name a segment by naming the two **endpoints** of the segment. If we wish to refer only to the portion of the line between points *A* and *B*, we would say "segment *AB*" (or "segment *BA*").

We name a ray by naming its endpoint first and then another point on the ray. This figure is "ray *AB*," but it is not "ray *BA*."



Instead of writing the word *line, segment,* or *ray,* we may draw a line, segment, or ray above the letters used to name the figure, as shown in this table:

	Figure	Name	Abbreviation
A	B	line AB	AB
A •	B	segment AB	ĀB
A •	B	ray AB	ĀB

Naming Lines, Segments, and Rays

When writing abbreviations for lines, segments, and rays, it is important to draw the figure above the letter pair. Segment  $\overline{AB}$ and  $\overline{AB}$  both name a segment, but AB (without the word *segment* in front or the bar above) means "the distance from A to *B*." So  $\overline{AB}$  refers to a segment, and AB refers to the *length* of the segment.

We may name an angle using the single letter at its vertex if there is no chance for confusion. Angle A in the figure below is the acute angle with A as its vertex.



Referring to angle *D*, however, would be unclear because there is more than one angle at *D*. In situations like this, we use three letters, with the vertex letter listed second. The obtuse angle at *D* is angle *ADC* (or angle *CDA*). The acute angle at *D* is angle *CDE* (or angle *EDC*). The straight angle at *D* is angle *ADE* (or angle *EDA*). We use the symbol  $\angle$  to abbreviate the word *angle*, so angle *ADC* may also be written as  $\angle ADC$ .

Example	e 1)
Math Symbols	In rectangle <i>ABCD</i> , name the segments
We can use symbols to identify	segments in the rectangle.
lines or line segments, such as $\overline{DC} \perp CB$ , and	both $\overline{AD}$ (or $\overline{DA}$ ) and $\overline{BC}$ (or $\overline{CB}$ ) are perpendicular to $\overline{AB}$ .
parallel lines or line segments, such as DA    CB.	Every rectangle has two pairs of parallel sides, so $\overline{AD}$ is parallel to $\overline{BC}$ and $\overline{BA}$ is parallel to $\overline{CD}$ .



**\*1.** The tallest teacher at Lincoln School is 6 feet 3 inches tall. A person who is 6 feet 3 inches tall is how many inches tall?

Written Practice

Distributed and Integrated

**\*2.** One sixth of the class was absent. What percent of the class was absent? What fraction of the class was present?

**3.** Rhode Island became the thirteenth state in 1790. Alaska and Hawaii became the forty-ninth and fiftieth states in 1959. How many years were there from 1790 to 1959?

**4. Represent** Write the standard form for  $(7 \times 1000) + (4 \times 10)$ .

**5. Estimate** Round 56 and 23 to the nearest ten. Multiply the rounded numbers. What is their product?

<b>*6.</b>	Multiple	Choice W	hich o	f these fract	ions	does not equ	al $\frac{1}{2}$ ?
(23)	A <u>6</u> 12	В	<u>12</u> 24	С	<u>24</u> 48	D	<u>48</u> 98

**7.** List Which factors of 12 are also factors of 16? (25)

**\*8. Analyze** A stop sign has the shape of a regular octagon. The sides of some stop signs are 12 inches long. What is the perimeter of a regular octagon with sides 12 inches long?

*9. 1 $-\frac{1}{5}$	* <b>10.</b> 1 $-\frac{3}{4}$	<b>11.</b> $3\frac{3}{3} - 1\frac{2}{3}$
* <b>12.</b> $\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10}$	* <b>13.</b> $5\frac{3}{4} + 4\frac{1}{4}$	
<b>14.</b> 4263 (14) <u>– q</u> 1784	<b>15.</b> \$19.34 + <u>m</u> \$50.00	<b>16.</b> 58 39 24 16 52 + 11



\*28. Oregon became a state in 1859, which was 4 years before West Virginia became a state. Arizona became a state 49 years after West Virginia. In what year did Arizona become a state? \* 29. (31) Estimate An event organizer must seat 65 guests at 3 long tables. If possible, the same number of guests are to be seated at each table. What is a reasonable estimate of the number of guests that will be seated at each table? Explain your answer.

\*30. Interpret Refer to the information and line plot below to answer parts a-e.

Rodric takes piano lessons. He records how many days he practices the piano each month and displays the data in a line plot. Here is Rodric's line plot for one full year:



- a. How many months did he practice more than 20 days?
- b. How many months did he practice between 15 and 20 days?
- c. What is the mode?
- d. What is the median?
- e. What is the range?



Main Street is a straight street and it intersects 3rd, 4th, and 5th Streets, which are all parallel to each other. Oak and Cedar Streets are perpendicular to 5th Street. Draw a map showing a possible arrangement of the six streets.



## Estimating Arithmetic Answers with Rounded and Compatible Numbers





#### Thinking Skill

#### Connect

We use rules to round numbers. What are the rounding rules? We have used arithmetic to find exact answers. For some problems, finding an exact answer may take many steps and a good deal of time. In this lesson we will practice quickly "getting close" to an exact answer. Recall that trying to get close to an exact answer is called *estimating*. When we estimate, we use rounded numbers, or compatible numbers, to make the arithmetic easier. We may even do the arithmetic mentally. An estimated answer is not an exact answer, but it is close to an exact answer.

Estimating can help reduce errors by showing us when a calculated answer is far from the correct answer. In other words, estimating can help us tell whether our calculated answer is *reasonable*.

#### **Example 1**

# Estimate the product of 29 and 19. Is the estimate greater than or less than the actual product? How do you know?

We estimate to quickly find *about* how much an exact answer would be. To estimate, we round the numbers *before* we do the work. The numbers 29 and 19 round to 30 and 20, which we can multiply mentally. Our estimated answer is **600.** Since **both 29 and 19 were rounded up** to greater numbers to make the estimate, we know that **600 is greater than the actual product.** 

**Verify** To which place was each number rounded?

#### **Example 2**

# Estimate the sum of \$8.95, \$7.23, \$11.42, and \$6.89 by rounding to the nearest dollar before adding.

For each amount, if the number of cents is 50 or more, we round to the next dollar. If the number of cents is less than 50, we round down.

**\$**9 + **\$**7 + **\$**11 + **\$**7 = **\$34** 

**Analyze** Which place did we look at to decide if we should round up to the next whole dollar or drop the cents?



**1.** Mrs. Nguyen made 6 dozen fruit cups for the party. The guests ate all <sup>(49)</sup> but 20 fruit cups. How many fruit cups were eaten?

Distributed and Integrated

**2. Explain** A millennium is 1000 years. A millennium is how many centuries? Explain how you know.

**\*3. Analyze** If water is poured from glass to glass until the amount of water in each glass is the same, how many ounces of water will be in each glass?



\*4. **Represent** Draw a circle and shade one third of it. What fraction of (Inv. 3, 97) the circle is *not* shaded? What percent of the circle is *not* shaded?

**\*5.** Estimate the product of 39 and 41.

Written Practice

 \*6.
  $1 - \frac{1}{10}$  \*7.
  $1 - \frac{3}{8}$  8.
  $4\frac{4}{4} - 2\frac{3}{4}$  

 \*9.
  $3\frac{1}{3} + 1\frac{2}{3}$  10.
  $6\frac{10}{10} - \frac{1}{10}$  \*11.
  $8 - 7\frac{\Box}{6}$ 

\* **12.** Estimate the sum of 586 and 317 by rounding both numbers to the nearest hundred before adding.

* <b>13.</b> (6)	89,786 26,428 57,814 + 91,875	* <b>14.</b> \$35,042 ( <sup>g)</sup> - \$17,651	* <b>15.</b> (55)	428 × 396
<b>16.</b> (26)	5y = 4735	17. 8 $ imes$ 43 $ imes$ 602 (18, 56)		

\* **18.** Divide:  $\frac{15}{8}$ . Write the quotient with a fraction.

398



**\* 19.** 967 ÷ 60 (54)

\*21. a. Name a fraction equal to  $\frac{1}{2}$ ?

**b.** Name a fraction that is less than  $\frac{1}{2}$ ?

**c.** Name a fraction that is greater than  $\frac{1}{2}$ ?

**22.** \$100 - (\$24 + \$43.89 + \$8.67 + \$0.98)

Multiple Choice The perimeter of this square is how (44, 53) many millimeters?

**A** 15 mm **B** 40 mm **C** 60 mm **D** 100 mm

**Explain** Think of an even number. Multiply it by 5. What number is 24. (2,15) the last digit of the product? Explain why.

**25.** Sonya is taking a trip. The clock at the right shows the time that her flight will leave tomorrow morning. If the flight takes 3 hours 20 minutes, what time will the plane land?

**\*26.** Refer to the spinner to answer parts **a–c.** 

- **a.** What are all the possible outcomes?
- b. What fraction names the probability that with one spin the spinner will stop on sector 3?
- **c.** What fraction names the probability that with one spin the spinner will stop on sector 1?
- \*27. Conclude Which angle in this figure appears to be an (61) obtuse angle?









\*28. Estimate How can you estimate the perimeter of this rectangle?

126 in.

32 in.

- **29. Analyze** Half of 100 is 50, and half of 50 is 25. What number is half of 25?
- \*30. The time in New York City is one hour ahead of the time in Chicago. (49) The time in Los Angeles is two hours behind the time in Chicago. If it is noon in New York City, what time is it in Los Angeles?



The average heart rate for a person is about 72 heartbeats per minute. Estimate how many times your heart beats in one hour. Use your estimate to find about how many times your heart beats in one day.



## Subtracting a Fraction from a Whole Number Greater Than 1



facts	Power Up F		
equivalent fractions	The following fractions are equal to one half: $\frac{1}{2}$ , $\frac{2}{4}$ , $\frac{3}{6}$ , $\frac{4}{8}$ . Read the fractions aloud and continue the pattern to $\frac{12}{24}$ .		
mental math	<b>a. Fractional Parts:</b> One fifth of 6 is $1\frac{1}{5}$ . How much is $\frac{1}{5}$ of 7? $\frac{1}{5}$ of 8?		
	<b>b.</b> Number Sense: A small pizza was cut into six equal slices. Margo ate one of the slices. What fraction of the pizza is left? ( <i>Think:</i> $1 - \frac{1}{6}$ .)		
	<b>c.</b> Number Sense: A large pizza was cut into ten equal slices. Tonya ate one of the slices. What fraction of the pizza is left? ( <i>Think</i> : $1 - \frac{1}{10}$ .)		
	d. Percent: The sale price is 10% off the regular price. How much is 10% of \$200?		
	e. Percent: The customer left a 20% tip for a \$50 order. How much is 20% of \$50?		
	f. Estimation: Rae read for 86 minutes and then watched television for 27 minutes. Round each measurement to the nearest ten minutes and then add.		
	<b>g. Probability:</b> Which is more likely: a coin landing heads up or a standard number cube landing on 2?		
	<b>h. Calculation:</b> $500 \div 10, \div 2, + 5, \div 5, + 3, \div 3$		
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Naomi has two dot cubes. Each cube has sides with a number of dots 1 through 6. Naomi plans to roll the two cubes and to add the numbers that appear on top. What are the possible totals that Naomi can roll with two cubes? Explain your reasoning.		



Recall that when we subtract a fraction from 1, we change the 1 to a fraction name for 1. If the problem is  $1 - \frac{1}{3}$ , we change the 1 to  $\frac{3}{3}$  so that the denominators will be the same. Then we can subtract.

We change this form:  $1 - \frac{1}{3}$ ... to this form:  $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$ 

In this lesson we will subtract fractions from whole numbers greater than 1.

Imagine we have 4 whole pies on a shelf. If someone asks for half a pie, we would have to cut one of the whole pies into 2 halves. Before removing half a pie from the pan, we would have 4 pies, but we could call those pies " $3\frac{2}{2}$  pies."



We use this idea to subtract a fraction from a whole number. We take 1 from the whole number and write it as a fraction with the same denominator as the fraction being subtracted. We will answer the problem  $4 - \frac{1}{2}$  to show how we do this.

> We change this form:  $4 - \frac{1}{2}$ ... to this form:  $3\frac{2}{2} - \frac{1}{2} = 3\frac{1}{2}$

Example 1 Name the number of shade

Name the number of shaded circles as a whole number and as a mixed number.

We see 3 whole circles.

Since one of the circles is divided into fourths, we can also say that there are two whole circles and four fourths of a circle, which we write as the mixed number  $2\frac{4}{4}$ .

**Analyze** How many fourths represent  $2\frac{4}{4}$ ? Explain how you know.



Distributed and Integrated

- 1. A 100-centimeter stick broke into 3 pieces. One piece was 7 centimeters long, and another was 34 centimeters long. How long was the third piece?
- \*2. K'Lyn had a 6-inch piece of ribbon. She used  $1\frac{1}{2}$  inches of the ribbon for a craft project. How long is the piece of ribbon she has left?
- 3. Isabel can make 4 quarter-pound hamburgers from 1 pound of meat. How many quarter-pound hamburgers can she make from 5 pounds of meat?
- \*4. (Analyze) In the 4 stacks of math books, there are 18, 19, 24, and 23 books. If the number of books in each stack is made the same, how many books will be in each stack?

**\*5. Estimate** Find the sum of 398 and 487 by rounding both numbers to the nearest hundred before adding.

**6.** List Which factors of 14 are also factors of 21?

\*7. Multiple Choice The distance around the earth is about 25,000 miles
 <sup>(53)</sup> at the equator. This distance is like which measurement of a circle?
 A radius
 B diameter
 C circumference
 D area

\*8. **Represent** What is the sum of five million, two hundred eightyfour thousand and six million, nine hundred eighteen thousand, five hundred?

<b>*9.</b> $7 - \frac{1}{3}$	* <b>10.</b> $6 - 2\frac{1}{2}$	* <b>11.</b> 8 $- 3\frac{3}{4}$
<b>* 12.</b> $\frac{8}{9} + \left(\frac{2}{9} - \frac{1}{9}\right)$	<b>* 13.</b> $5\frac{3}{4} - \left(3\frac{2}{4} + 1\frac{1}{4}\right)$	
<b>14.</b> 43,716 <sup>(9)</sup> - 19,537	* <b>15.</b> \$6.87 (55) × 794	<b>16.</b> $\frac{\$14.72}{8}$

**\*17.** Divide:  $\frac{20}{9}$ . Write the quotient with a fraction.

**18.** 20)951 **19.** 50)2560

**20.**  $50 \times (400 + 400)$  **21.**  $(400 + 400) \div 40$ 

**22.** 4736 + 2849 + 351 + 78

\*23. If three eighths of the class was not in the library, what fraction of the (Inv. 3, 60) class was in the library? What percent of the class was in the library?

**24.** (Analyze) these fractions in order from least to greatest. (*Hint:* Decide whether each fraction is less than, equal to, or greater than  $\frac{1}{2}$ .)

$$\frac{5}{10}, \frac{5}{8}, \frac{5}{12}$$

\*25. Analyze What is the perimeter of this equilateral triangle?



**\* 26.** Use this pictograph to answer the questions that follow: (Iny, 5)

Animal	Average Weight (in pounds)
Horse	tor foor foor foor foor foor foor foor
Chimpanzee	100 10
Saltwater Crocodile	100 100 100 100 100 100 100 100 100 100
Gorilla	100 100 100 100 10
	Key: 🙀 = 100 pounds

- **a.** Write the average weight of each animal, and order the weights from least to greatest.
- **b.** About how many times greater is the average weight of a gorilla than the average weight of a chimpanzee?
- c. Name the two animals that together weigh about 1 ton.
- \* 27. The average body temperature of a polar bear is about 99°F. The average body temperature of an arctic gull is about 93°F. About how many degrees warmer is a polar bear than an arctic gull?
- **\*28.** A school day at Tyra's school ends at 3:10 p.m. What kind of angle is formed by the minute and hour hands of a clock at that time?
  - **29.** One way to estimate the quotient of  $350 \div 4$  is to round 350 to 400 and find the quotient of  $400 \div 4$ . Describe a way to make a closer estimate of  $350 \div 4$ .
- \*30. About 39% of the world's energy is produced by petroleum, about 24% is produced from coal, and about 23% is produced from natural gas. What percent of the world's energy is produced from these three fuels?



Power Up

# Using Money to Model Decimal Numbers

facts	Power Up F		
equivalent fractions	The following fractions are equal to one half: $\frac{1}{2}$ , $\frac{2}{4}$ , $\frac{3}{6}$ . Read the fractions aloud and continue the pattern to $\frac{12}{24}$ .		
mental math	<b>a.</b> Powers/Roots: The symbol $\sqrt{}$ is a square root symbol. We read $\sqrt{25}$ as "the square root of 25." The expression $\sqrt{25}$ equals 5 because $5 \times 5 = 25$ . What does $\sqrt{49}$ equal?		
	<b>b. Number Sense:</b> $1-\frac{2}{3}$		
	c. Number Sense: $1-\frac{3}{4}$		
	<b>d. Number Sense:</b> $1-\frac{4}{5}$		
	e. Measurement: Lisa cut the 250-cm string into ten equal pieces. How long was each piece? ( <i>Think:</i> 250 cm ÷ 10.)		
	f. Estimation: T'Rae threw the baseball 55 feet 8 inches to the catcher. Round this distance to the nearest foot.		
	g. Probability: If the chance of rain is 30%, what is the chance it will not rain?		
	<b>h. Calculation:</b> 50% of 60, $+$ 10, $\div$ 5, $+$ 2, $\times$ 10		
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Two figures are similar if they have the same shape. These two triangles are not similar. Draw a triangle that is similar to the top triangle and that has a perimeter of $4\frac{1}{2}$ inches. What length will you use for each side of the triangle?		



#### Thinking Skill

Connect

Describe the relationship between the ones place and the thousands place. In this lesson we will use money to illustrate decimal numbers. Recall that in our number system the position a digit occupies in a number has a value, called *place value*.



As we move to the left from the ones place, the value of each place is ten times greater than the place to its right. We have shown the value of four places, but the pattern continues without end.

Notice that as we move in the other direction (to the right), the value of each place is one tenth the value of the place to its left.



This pattern also continues without end. To the right of the ones place is the tenths place, the hundredths place, the thousandths place, and so on. These places are called **decimal places**. In this diagram we show the first three decimal places:



Notice the decimal point between the ones place and the tenths place. We use a decimal point as a reference point, like a landmark, so that we know where the whole-number places end and the decimal places begin. We do not need to use the decimal point to write whole numbers because it is understood that in whole numbers, the digit farthest to the right is in the ones place. One use of decimal numbers is to write dollars and cents, such as \$6.25. This collection of bills and coins totals \$6.25:



Notice that the number of bills and coins matches the digits in \$6.25: 6 ones, 2 dimes, 5 pennies. We can use pennies, dimes, dollars, \$10 bills, and \$100 bills to model place value.

Place Name	hundreds	tens	ones	tenths	hundredths
Place Value	100	10	1	<u>1</u> 10	<u>1</u> 100
Place				•	
Money Value of Place	\$100 bills	\$10 bills	\$1 bills	dimes	pennies

Place-Value Chart

The last row of the chart gives the money value of each place. The first place to the right of the decimal point is the tenths place. Since a dime is one tenth of a dollar, we may think of this as the dimes place. The second place to the right of the decimal point is the hundredths place. Since a penny is one hundredth of a dollar, we may think of this as the pennies place.

A dime demonstrates the value relationship between adjoining places. While a dime is ten times the value of a penny (thereby making it worth 10 cents), it is also one tenth the value of a dollar.



**Connect** Describe the relationship between the tenths place and the ones place as shown in the second row of the chart.

Example	1
	What combination of dollars, dimes, and pennies makes \$4.65 using the fewest bills and coins possible?
	The digits in the number \$4.65 show us how many of each bill or coin to use. We need <b>4 dollars, 6 dimes, and 5 pennies.</b>

	(We would probably use two quarters, a dime, and a nickel to make
	65 cents with actual money, but we do not use quarters and nickels to model decimal place value.)
Example 2	2
	What is the place value of the 4 in \$6.24?
	The 4 is in the second place to the right of the decimal point, which is the <b>hundredths</b> place. This is reasonable because 4 shows the number of pennies, and a penny is a hundredth of a dollar.
Example 3	3
	Is \$3.67 closer to \$3.60 or to \$3.70?
	To answer this question, we round \$3.67 to the nearest ten cents, that is, to the tenths place. Since 7 cents is more than half a dime, \$3.67 rounds up to <b>\$3.70.</b>
	<b>Connect</b> Between which two whole numbers on a number line is 3.70?
Example 4	
	On his way home from work, T' Vaughn stopped at the supermarket and purchased carrots for \$1.66, broccoli for \$3.42, and cauliflower for \$2.55. What is a reasonable estimate of the cost of the vegetables?
	Instead of rounding to the nearest dollar, we will use compatible numbers and round to the nearest half dollar.
	\$1.50 + \$3.50 + \$2.50 = <b>\$7.50</b>
Lesson Practice	What is the place value of the 5 in each of these numbers?
	<b>a.</b> \$25.60 <b>b.</b> \$54.32 <b>c.</b> \$12.75 <b>d.</b> \$21.50
	<ul> <li>e. What combination of dollars, dimes, and pennies makes</li> <li>\$3.84 using the fewest bills and coins possible?</li> </ul>
	f. Is \$12.63 closer to \$12.60 or to \$12.70?
	<b>g.</b> Is \$6.08 closer to \$6.00 or to \$6.10?
	h. <b>Estimate</b> Teresa purchased three notebooks for \$1.49 each. What is a reasonable estimate of the total cost of the notebooks? Explain your answer.

Distributed and Integrated

- \*1. What is the sum of one hundred sixteen thousand, five hundred (52) twenty-one and two hundred fifty-three thousand, four hundred seventy-nine?
- **2.** At the annual clearance sale, *Shutter Shop* lowered the price of all its cameras. Aliana wants to buy a new camera that costs \$30.63. She has \$17.85. How much more money does she need?
- **\*3.** In the auditorium there were 30 rows of seats with 16 seats in each row. <sup>(49)</sup> If there were 21 empty seats, how many seats were filled?
- **4.** A home-improvement company estimates that it will take 324 hours to remodel a house. If that amount of work is to be completed equally by 6 employees, what number of hours will each employee be expected to work?
- **\*5.** Estimate the product of 68 and 52.

Written Practice

- **\*6.** If three tenths of the bowling pins were up, what fraction of the bowling <sup>(*lnv. 2, 60*)</sup> pins were down? What percent of the bowling pins were down?
- **\*7.** Numbers written in dollars and cents (such as \$54.63) have how many decimal places?
- **\*8.** What combination of dollars, dimes, and pennies makes \$3.25 using the fewest bills and coins possible?
- **\*9.** Is \$4.82 closer to \$4.80 or to \$4.90?
- \* **10.** Divide 25 by 8. Write the quotient with a fraction. (58)
- \* **11.** List Which factors of 20 are also factors of 30?
  - **12.** Geraldo stayed up late on Friday night, and on Saturday morning he slept until  $1\frac{1}{2}$  hours before noon. What time did Geraldo wake up on Saturday morning?

<b>13.</b> (14)	360 <i>- a</i> = 153	<b>14.</b> 5 <i>m</i> = 875	<b>*15.</b> $\frac{3}{5} + f = 1$
<b>*16.</b> (59)	$\frac{5}{5}-z=\frac{3}{3}$	<b>17.</b> \$30.48 ÷ 6	<b>18.</b> 60) 1586
<b>19.</b> (13)	\$4.34 \$0.26 \$5.58 \$9.47 \$6.23 + \$0.65	<b>20.</b> 5 × 4 × 3 × 2 ×	1 × 0
* <b>21.</b> (63)	$7 \\ -3\frac{2}{3}$	*22. $1\frac{1}{3}$ + $2\frac{2}{3}$	*23. 4 $-3\frac{3}{4}$

**\*24.** Figure *PQRST* is a regular pentagon. If  $\overline{PQ}$  measures 12 mm, then what is the perimeter of the pentagon?



**25. a.** When 10 is divided by 3, what is the remainder? (22, 26)

**b.** When 100 is divided by 3, what is the remainder?

**\*26.** What is the perimeter of this equilateral triangle?



**\*27.** Suppose the 7 letter tiles below are turned over and mixed up. <sup>(57)</sup> Then one tile is selected.

J В F С Μ R Τ

What is the probability that the letter selected will follow Q in the alphabet?

\*28. Several temperatures that were recorded one April day in Hershey, <sup>(Inv. 5)</sup> Pennsylvania, are shown in the table. Display the data in a line graph.

Time	Temperature (°F)
12:00 a.m.	43
3:00 a.m.	46
6:00 a.m.	45
9:00 a.m.	48
12:00 p.m.	58

- \*29. Tuesday's high temperature was 12 degrees cooler than Monday's and
   <sup>(49)</sup> 7 degrees cooler than Wednesday's. If Wednesday's high temperature was 59°, then what was Monday's high?
- \*30. The coach of a hockey team has purchased six new hockey pucks at a cost of \$3.49 each. Explain how addition can be used to make a reasonable estimate of the total cost.



Jason earns extra money after school and on Saturdays by doing yard work. He earned \$25.45 in September, \$35.25 in October, and \$29.75 in November. Use compatible numbers to make a reasonable estimate of the amount of money Jason earned altogether, and then explain your answer.



## Decimal Parts of a Meter

Power Up

Power Up C

mental math

facts

- a. Percent: 10% of 10
  b. Percent: 10% of 100
  - **c. Percent:** 10% of 1000
  - **d. Fractional Parts:** One third of 11 is  $3\frac{2}{3}$ . How much is  $\frac{1}{3}$  of 13? ...  $\frac{1}{3}$  of 14?
  - e. Powers/Roots:  $\sqrt{36}$
  - f. Number Sense:  $1 \frac{2}{5}$
  - **g. Measurement:** The door is 6 feet 7 inches tall. How many inches is that?
  - h. Calculation:  $8 \times 5$ , -10,  $\div 5$ ,  $\times 7$ , -2,  $\div 5$

problemGina, Bryce, and Shelley collected donations for the team trip.solvingGina collected \$53.38. Bryce collected \$22.89 less than Gina.Altogether, the three students collected \$123.58. Find the<br/>amounts that Bryce and Shelley collected.

#### Focus Strategies: Make It Simpler; Write an Equation

**Understand** We are told that Gina, Bryce, and Shelley collected donations. We know how much Gina collected, the difference between Gina's and Bryce's amounts, and the total collected by all three. We are asked to find the amounts collected by Bryce and Shelley.

**Plan** The decimal places in the money amounts might be distracting, so we will *make the problem simpler* to more easily see a solution process. Then we will *write equations* to find each amount.

**Solve** We round the given amounts to the nearest ten dollars and perform a "trial solution." Gina collected about \$50. Bryce collected about \$20 less than Gina, so he collected about \$50 – \$20, or about \$30. The total collection was about \$120. We subtract Gina's and Bryce's amounts from the total to find the approximate amount collected by Shelley:

We know that our solution process makes sense if we add the three approximate amounts for Gina, Bryce, and Shelley:

50 + 30 + 40 = 120

Now we will use exact numbers in our solution. We can use a calculator to speed the process. First we calculate the amount Bryce collected:

\$53.38 - \$22.89 = **\$30.49** 

Now we subtract the amounts for Gina and Bryce from the total to find the amount for Shelley:

\$123.58 - \$53.38 - \$30.49 = **\$39.71** 

**Check** We know that our answers are reasonable because they are close to the rounded numbers we used when solving a simpler problem. Also, we find that the two amounts we found plus the amount we are given total \$123.58, which equals the total given in the problem.

**New Concept** 

#### **Math Language**

The metric system of measurement is a base-ten system, and it is similar to our monetary system and to our numeration system. In this lesson we will use metric units of length to model decimal numbers. The basic unit of length in the metric system is a **meter**. One big step is about a meter long. Many school classrooms are about 10 meters long and 10 meters wide.

Meters may be divided into tenths, hundredths, and thousandths. These smaller units are *decimeters, centimeters,* and *millimeters.* 



A **decimeter** is one tenth of a meter. A **centimeter** is one tenth of a decimeter and one hundredth of a meter. A **millimeter** is one tenth of a centimeter and one thousandth of a meter. These fractional parts of a meter can represent the first three decimal places.



	Activity 2
	Measuring with a Meterstick
	Material needed:
	meterstick
	Use a meterstick to measure the following classroom items.
	Use the measurements to help you answer the questions below.
	Item 1: height of the door
	Item 2: width of the door
	Item 3: height of desk
	Item 4: length of bulletin board
	Item 5: length of math book
	a. Is the measurement more than or less than a meter?
	<b>b.</b> What is the measurement in meters? Use a decimal number
	to write the measurement in meters.
Lesson	<b>a. Multiple Choice</b> Which of these is the most reasonable
	measurement for the length of an automobile?
	A 4.5 meters B 4.5 decimeters
	<b>C</b> 4.5 centimeters <b>D</b> 4.5 millimeters
	b. Alonso is 1 meter plus 43 centimeters tall. Use a decimal number to write Alonso's height in meters.
	c. A ruler is about 30 centimeters long. About how many decimeters long is a ruler?
Written Prac	Distributed and Integrated
*1. Represent D	raw a quadrilateral with one pair of horizontal segments

and one pair of vertical segments.

\*2. Analyze The players are divided into 10 teams with 12 players on each team. If all the players are divided into 8 equal teams instead of 10, then how many players will be on each team?

**3.** Below is a representation of a rectangular field that is 100 yards long <sup>53)</sup> and 40 yards wide. Use a formula to find the perimeter of the field.



- **4.** A yard is 36 inches. How many inches is one fourth of a yard? One (*lnv. 2,* 46) fourth of a yard is what percent of a yard?
- **5.** Pilar's school starts at 8:30 a.m. If it is now 7:45 a.m., how many minutes does she have until school starts?
- **\*6. Estimate** Find the sum of 672 and 830 by rounding to the nearest hundred before adding.
- 7. a. What fraction of the rectangle is shaded?
  - **b.** What fraction of the rectangle is not shaded?
  - c. Explain how you know your answers are reasonable.
- **\*8.** The refrigerator was 1 meter plus 32 centimeters tall, and it was <sup>(65)</sup> 82 centimeters wide. Write the height of the refrigerator in meters.
- \*9. Half a meter is how many decimeters?
- \* **10. Analyze** Arrange these fractions in order from least to greatest. (*Hint:* <sup>(23)</sup> Decide whether each fraction is less than, equal to, or greater than  $\frac{1}{2}$ .)
  - $\frac{4}{4}, \frac{3}{8}, \frac{2}{3}, \frac{5}{10}$
  - **11.** The number 9 has three different factors. The number 10 has how many different factors?
- \* **12.** Divide:  $\frac{15}{4}$ . Write the quotient as a mixed number.
  - **13.** Write the greatest odd number that uses the digits 3, 4, and 5 once each.
- **14.** Five hundred is how much more than three hundred ninety-five?



<b>15.</b>	36,195	<b>16.</b>	41,026	<b>* 17.</b>	608
(0)	17,436	(0)	- 39,543	(00)	$\times 479$
	+ 42,374				
18.	2637 ÷ 4	19.	40)\$33.60	20.	3360
(26)		(54)		(54)	20
	-3 -5		3		.3 .
<b>* 21.</b> (59)	$3\frac{3}{8} + 5\frac{3}{8}$	<b>* 22.</b> (63)	$5-3\frac{3}{8}$	<b>23.</b> (43)	$3\frac{3}{4} - 3$
<b>*24.</b>	$6 \times 42 \times 2$	20 <b>25.</b>	\$20 - (\$5.63 + \$12)		
(10,00)		(10) = 1)			
26.	Analyze	To find the number	of eggs in $2\frac{1}{2}$ dozen, Kel	lan tho	ought of
(23, 51)	$2\frac{1}{2}$ as $(2 + $	$\frac{1}{2}$ ). Then he used th	ne Distributive Property.		
		$2^{\frac{1}{2}}$ dozen = 2	$2 dozen + \frac{1}{2} dozen$		
		2 402011	2 402011 + 2 402011		
	How many	eggs is $2\frac{1}{2}$ dozen?			
<b>27.</b>	By which o	f these numbers is	1080 divisible?		
(22, 42)		0.0	5 6 9 10		
		۷, ۵,	J, U, J, IU		

Refer to the spinner to answer problems 28-30.

- **\*28. a.** What is the probability that with one spin the outcome will be an even number?
  - **b.** What is the probability that with one spin the outcome will be a number less than 4?
  - **c.** What is the probability that with one spin the outcome will be a number less than 5?
  - **29.** The grassland habitat of a red kangaroo has an average temperature of about 84°F. The desert habitat of a rattlesnake has an average temperature of about 104°F. How many degrees cooler is the habitat of the red kangaroo?
- \***30.** To prepare for a test, Shaun studied for half as long as Adolfo. Adolfo studied for half as long as J'Ron. J'Ron studied for 60 minutes. How long did Shaun study?

1

3

2

 $\geq$ 

4



Power Up

## Reading a Centimeter Scale

facts	Power Up F
estimation	Hold your fingers a decimeter apart a centimeter apart a millimeter apart.
mental math	<b>a. Fractional Parts:</b> One fifth of 11 is $2\frac{1}{5}$ . How much is $\frac{1}{5}$ of 16? $\dots \frac{1}{5}$ of 17?
	<b>b. Time:</b> What time is 1 hour 20 minutes after 11:10 p.m.?
	c. Measurement: How many ounces equal a pound?
	<b>d. Estimation:</b> Pam ran once around the block in 248 seconds. What is 248 seconds to the nearest minute?
	e. Powers/Roots: $\sqrt{9}$
	f. Number Sense: $1 - \frac{3}{10}$
	g. Number Sense: $6 \times 23$
	<b>h. Calculation:</b> 25% of 16, $\times$ 6, $+$ 6, $\div$ 6, $\times$ 2, $\div$ 10
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Risa stacked some small cubes together to form this larger cube. How many small cubes did Risa use? Explain how you arrived at your answer.
New Concept	
	In this lesson we will measure objects using a centimeter ruler

In this lesson we will measure objects using a centimeter ruler. A ruler is usually 30 centimeters long and is further divided into millimeters. Each millimeter is one tenth of a centimeter. Here we show part of a centimeter ruler:

in	<u>.</u>	шппп		huuluu	huluu	hulluu			huuluu	րուրուլ
cm	1	2	3	4	5	6	7	8	9	10

#### Thinking Skill

#### Generalize

About 2.5 centimeters are equal to 1 inch. About how many centimeters are equal to 12 inches? Explain how you know. In Lesson 65 we learned to write lengths as decimal parts of a meter. For example, fifteen centimeters can be written as 0.15 m, which means "15 hundredths of a meter." To show the units as centimeters rather than meters, we would write fifteen centimeters without a decimal point (15 cm), changing the units from "m" to "cm." (Similarly, we write fifteen cents as 15¢ instead of \$0.15 if the units are cents instead of dollars.)

How we write a particular length depends upon whether we use millimeters, centimeters, or meters as units. This segment is 15 millimeters long:

mm	10	20	30	40	50	7
սուր	ահա	հուվոս	huuluu	huuluu	huuluu	Т)
		-				
h.	mhuu		huuluuu	աղոո	huuluu	T)

The segment is also 1.5 centimeters long. The tick marks on the centimeter scale divide each centimeter into ten equal lengths that are each  $\frac{1}{10}$  of a centimeter. The end of the segment is 5 lengths past the 1 centimeter mark.



#### Write the length of this segment

- a. as a number of millimeters.
- b. as a number of centimeters.
- a. The length of the segment is 24 mm.
- b. The length of the segment is 2.4 cm.



Just as tenths of a centimeter can be written as a decimal number, so can tenths on a number line. Here we show a number line with the distance between whole numbers divided into tenths. We show the decimal numbers represented by four points on the number line.



**Verify** Name the fraction or mixed number that each arrow is pointing to.



**2.** Use the table to answer the questions that follow.

Number of Octopuses	1	2	3	4
Number of Legs	8	16	24	32

- **a. Generalize** Write a rule that describes how to find the number of legs for any number of octopuses.
- b. **Predict** How many legs would 20 octopuses have?
- **3. Estimate** Find the difference of 794 and 312 by rounding both numbers to the nearest hundred before subtracting.
- \*4. Tomas could carry 6 containers at one time. If 4 containers weighed 20 pounds, then how much did 6 containers weigh?
- \*5. Analyze When one end of the seesaw is 9 inches above the ground, the other end is 21 inches above the ground. How far are the ends above the ground when the seesaw is level?
- **6.** Compare:  $\frac{3}{5} \bigcirc \frac{4}{9}$ . (*Hint:* Decide whether each fraction is more than  $\frac{1}{2}$  or less than  $\frac{1}{2}$ .)
- **7.** Which digit in 4318 is in the same place as the 7 in 96,275?
- **8. a.** What fraction of this rectangle is shaded?
  - b. What fraction of the rectangle is not shaded?
- **\*9.** Find the length of this tack to the nearest tenth of a centimeter.
- \* **10.** Compare: 1 decimeter  $\bigcirc$  1 centimeter
- \* **11.** A pastry chef has 13 pounds of apple slices to make 8 apple pies. If the chef distributes the slices equally, what will be the weight of the apple slices used to make each pie?





- \*12. (Analyze) Ruby ran around the block. If the block is 200 yards long and 60 yards wide, how far did she run? You may draw a picture to solve the problem.
- \* **13.** Segment *AB* is 40 millimeters long. Segment *BC* is 35 millimeters long. How long is  $\overline{AC}$ ?

A	B •	C
<b>14.</b> 87,864 46,325 + 39,784	<b>15.</b> 34,125 (9) <u>- 16,086</u>	<b>16.</b> \$400.00 ( <sup>13)</sup> - \$398.57
<b>17.</b> $\frac{5628}{6}$	<b>18.</b> 807 <sup>(56)</sup> × 479	<b>19.</b> \$7.00 (29) × 800
<b>20.</b> $3\frac{2}{3} - \left(2\frac{1}{3} + 1\right)$	<b>21.</b> $4 - \left(2 + 1\frac{1}{4}\right)$	<b>22.</b> $36 \times 60 \times 7$
<b>23</b> $(20 - (28 + 20))$		

(13, 24)

\*24. Use this information to answer parts a and b: (16, Inv. 5)

There are 16 players on the Norwood softball team. Ten players are in the game at one time. The rest of the players are substitutes. The team won 7 of its first 10 games.

- a. The Norwood softball team has how many substitutes?
- **b.** Multiple Choice If the team played 12 games in all, what is the greatest number of games the team could have won? **A** 12 **C** 9 **B** 10 **D** 7

\*25. **Represent** Make a frequency table for the number of letters in (Inv. 5) the names of the twelve months of the year. "May" has the fewest (3 letters). "September" has the most (9 letters). The months of the year are listed below for reference.

> January, February, March, April, May, June, July, August, September, October, November, December

\*26. **Connect** To what decimal number is the arrow pointing?

9 10 11

- **27. Represent** Draw a right triangle.
- \*28. (62) **Estimate** A large city is home to more than 20 different museums. Pedro and Consuelo would like to visit all of the museums during their 7-day stay in the city. About how many museums would they need to visit each day to accomplish their goal? Explain your answer.
- \*29. (62) The land area of Nez Perce National Historical Park is about 2023 hectares. The land area of Cumberland Gap National Historic Park is about 10 times greater. What is a reasonable estimate of the land area of Cumberland Gap National Park? Explain why your estimate is reasonable.
- **30.** (10) Write a word problem for the equation n + 5 = 8. Then solve the equation.



Use a centimeter ruler to measure the length of five different objects in the classroom. Write the measurement of each object first in centimeters and then in millimeters. Then list the objects in order from longest to shortest.



## Writing Tenths and Hundredths as Decimal Numbers



facts	Power Up G				
equivalent fractions	The following fractions are equal to one half: $\frac{1}{2}$ , $\frac{2}{4}$ , $\frac{3}{6}$ . Read the fractions aloud and continue the pattern to $\frac{12}{24}$ .				
mental math	<ul> <li>a. Measurement: 10% of a decimeter is a</li> <li>b. Measurement: 10% of a centimeter is a</li> <li>c. Powers/Roots: √64</li> <li>d. Number Sense: 640 ÷ 20</li> <li>e. Geometry: An octagon has how many more sides than a pentagon?</li> <li>f. Time: How many days are in a leap year? in a common year?</li> <li>g. Probability: The answer choices on a test problem are labeled A, B, and C, which means the probability of guessing correctly is <sup>1</sup>/<sub>3</sub>. What is the probability of guessing incorrectly?</li> <li>h. Calculation: 6 × 8, -3, ÷ 5, × 3, +1, ÷ 4</li> </ul>				
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Angie is playing a board game with her niece, Amber. For each turn, a player rolls two dot cubes to determine how many spaces to move her game piece. Angie wants to move her piece 10 spaces. Make a table that shows the ways Angie can roll a total of 10 with two dot cubes.				

## New Concept

#### **Math Language**

A common fraction has a wholenumber numerator and a whole-number denominator. In this lesson we will write fractions with denominators of 10 or 100 as decimal numbers. A **common fraction** with a denominator of 10 can be written as a decimal number with one decimal place. The numerator of the common fraction is written in the tenths place of the decimal number. For example,

 $\frac{1}{10}$  can be written as 0.1

The common fraction  $\frac{1}{10}$  and the decimal number 0.1 are both named "one tenth" and are equal in value. The zero to the left of the decimal point shows that the whole-number part of the decimal number is zero.



#### **Example 3**

# Name the number of shaded circles as a mixed number and as a decimal number.



One whole circle is shaded, and one tenth of

another circle is shaded. We write one and one tenth as the mixed number  $1\frac{1}{10}$ . We write one and one tenth as a decimal number by writing the whole number and then the decimal fraction: **1.1**.

A common fraction with a denominator of 100 can be written as a decimal number with two digits after the decimal point. The digits of the numerator of the common fraction become the digits of the decimal number.

 $\frac{1}{100}$  is the same as 0.01

Notice that in the decimal number we placed the 1 *two places* to the right of the decimal point so that the 1 is in the *hundredths place*. Study these examples:

 $\frac{3}{100} = 0.03$   $\frac{30}{100} = 0.30$   $\frac{97}{100} = 0.97$ 

Notice that when the fraction has only one digit in the numerator, we still write two digits after the decimal point. In the first example above, we write the 3 in the hundredths place and a 0 in the tenths place.

#### Example 4 Write twelve hundredths as a common same digits fraction and as a decimal number. 12 Twelve hundredths is written as a common 100 fraction like this: $\frac{12}{100}$ . A common fraction with a denominator of 100 can be written as two places a decimal number with two digits after the decimal point. We write the decimal number twelve hundredths as 0.12. Example 5 Write $4\frac{3}{100}$ as a decimal number. We write the whole number, 4, to the left of the decimal point. To

write hundredths, we use the two places to the right of the decimal point. To point. The mixed number  $4\frac{3}{100}$  is equal to **4.03**.

#### **Thinking Skill**

#### Discuss

Name each coin as both a fractional part and a decimal part of a dollar:

- penny
- nickel
- dime
- quarter
- half-dollar

#### Example 6

Name the shaded portion of the square as a common fraction and as a decimal number.

Thirty-three of the hundred parts are shaded. The common fraction for thirty-three hundredths is  $\frac{33}{100}$ . The decimal number is **0.33**.



Compare the shaded squares found in Examples 2 and 6. Notice that more of the square is shaded to show 0.4 than to show 0.33. In the following activity you will compare decimal numbers by shading and comparing portions of squares.

## Activity

#### **Comparing Decimal Numbers**

Material needed:

• Lesson Activity 38

On **Lesson Activity 38**, shade the squares to represent each decimal number. Compare the decimal numbers by comparing the shaded part of each square.

Lesson Practice	a. Name the shaded portion of this rectangle as a fraction and as a decimal number.
	b. Name the unshaded portion of the rectangle as a fraction and as a decimal number.
	<b>c.</b> Name the number of shaded circles as a mixed number and as a decimal number.
	d. Name the shaded portion of the square as a fraction and as a decimal number.
	e. Name the unshaded portion of the square as a fraction and as a

decimal number.

Written Practice

Distributed and Integrated

- \*1. Analyze The books are divided into 4 stacks with 15 books in each stack. If the books are divided into 5 equal stacks instead of 4, how many books will be in each stack?
- **2.** A loop of string 20 inches long is made into the shape of a square. How long is each side of the square?
- **3.** Geneviève rented 2 movies for \$2.13 each. She paid for them with a \$10 bill. How much change did she receive?
- \*4. **Represent** Write the mixed number  $2\frac{3}{10}$  with words and as a decimal number.
  - **\*5. Represent** Write the fraction twenty-one hundredths as both a common fraction and as a decimal number.
  - \*6. **Represent** Write the fraction  $\frac{99}{100}$  as a decimal number.
  - **\*7.** Use a fraction and a decimal number to name the "<sup>(67)</sup> "unshaded" portion of this rectangle:
- **\*8.** Find the length of this segment in centimeters and in millimeters:









**\*9.** Name the shaded part of this square as a fraction and as a decimal number:

**10.** A greenhouse employee has 35 ounces of a potting soil and must place equal amounts of the soil into each of 8 small clay pots. What is the weight of the potting soil that should be placed in each clay pot?

**\* 11. Connect** Use a common fraction and a decimal number to name the point marked by the arrow.



- \*24. The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 are written on separate cards. The cards are then turned over and mixed up, and one card is selected.
  - a. What is the probability that the number on the card is 7?
  - b. What is the probability that the number on the card is odd?

- 25. Use this menu to answer parts a-c:
  - **a.** What is the total cost of one chicken salad and one small drink?
    - **b.** Peyton paid for 2 fruit salads with a \$10 bill. How much money should he get back?
    - **c.** Mr. Howard bought one of each type of salad for his family. About how much did he spend?

\*26. a. What type of polygon is figure ABCDEF?

- **b.** If this polygon is regular and the perimeter is 12 inches, then how long is each side?
- 27. Use the numbers in problem 26 to answer parts a-d.
  - **a.** What is the mode?
  - b. What is the median?
  - c. What is the range?
  - **d.** What is the average?
- **\*28.** A square with sides one decimeter long has a perimeter of how many centimeters?
  - 29. Explain One day students in a 45-minute math class spent
     8 minutes correcting homework and 18 minutes learning about a new concept. The remainder of the time was spent completing an activity. What length of time did the students spend that day completing an activity? Explain how you found the answer.

**\*30.** Multiple Choice Which of the following choices best describes your height?

- A between 1 and 2 meters
- **B** between 2 and 3 meters
- **C** more than 3 meters
- **D** less than 1 meter

Ivieriu
Shrimp Salad \$6.50
Chicken Salad \$5.25
Fruit Salad \$3.00
Orinks:
Regular \$0.80
Small\$0.50
Prices include sales tax.

Maria





 Naming Decimal Numbers

Power Up

facts	Power Up E							
mental	a. Powers/Roots: 81							
math	b. Number Sense: The cake was cut into 12 slices, and 5 slices have been eaten. What fraction of the cake remains?							
	c. Number Sense: $10 \times 10$							
	<b>d. Number Sense:</b> $10 \times 10 \times 10$							
	<b>e. Fractional Parts:</b> One tenth of 23 is $2\frac{3}{10}$ . How much is $\frac{1}{10}$ of 43? $\frac{1}{10}$ of 51?							
	<ul> <li>f. Estimation: Shaquille bought a pencil and a compass for \$3.52. He has \$6.78. If Shaquille used compatible numbers, approximately how much money would he have left?</li> </ul>							
	g. Probability: If the chance of rain is 60%, what is the chance it will not rain?							
	<b>h. Calculation:</b> Find 25% of 40, $+ 1, \times 3, -1, \div 4$							
problem solving	Choose an appropriate problem-solving strategy to solve this problem. To decide which homework assignment to work							
	on first, Jamie labeled 5 index cards as science READING							
	down, mix them up, and then draw one card. What is the probability she will choose a subject other than math?							

### **New Concept**

#### **Math Language**

The mill was first introduced in 1786 by the Continental Congress as a money amount worth  $\frac{1}{1000}$  of the federal dollar. Some states issued a token in this amount as a way to pay sales tax, but by 1960 the mill was no longer made. Today, the cost of gasoline is still represented in tenths of a cent. For example, \$3.019 per gallon is three dollars, one penny, and nine mills.

In this lesson we will name decimal numbers that have one, two, or three decimal places. The third place to the right of the decimal point is the thousandths place, and its value is  $\frac{1}{1000}$ . We do not have a coin for  $\frac{1}{1000}$ , but we do have a name for  $\frac{1}{1000}$ of a dollar. A thousandth of a dollar is a **mill.** Ten mills are equal to one penny.



To name a decimal number that has digits on both sides of the decimal point, we mentally break the number into two parts: the whole-number part and the fraction part. The whole-number part is to the left of the decimal point. The fraction part is to the right of the decimal point.

To read this decimal number: 12.5

we mentally break it into two parts, like this: (

2).(5

We read the whole-number part first, say "and" at the decimal point, and then read the fraction part. To read the fraction part, we read the digits as though they name a whole number. Then we say the place value of the last digit. The last digit of 12.5 is 5. It is in the tenths place.



We read other decimal numbers using the same process. To read the fraction part 6.12, read the digits after the decimal as a whole number and then say the place value of the last digit. The last digit of 6.12 is 2, and it is in the hundredths place.

(6) . (12)
 ↓ ↓
 ↓ ↓
 six and twelve hundredths





**Represent**) Use words to name each decimal number:

- **b.** 24.42
- **c.** 0.125
- **d.** 10.075

**Represent** Use digits to write each decimal number:

- e. twenty-five and fifty-two hundredths
- f. thirty and one tenth
- g. seven and eighty-nine hundredths
- h. two hundred thirty-four thousandths

A mill is  $\frac{1}{1000}$  of a dollar. Write the amounts in decimal word form, as a decimal, and as a fraction for the pictorial models below.



- **1.** It takes Keb 20 minutes to walk to school. What time should he leave for school if he wants to arrive at 8:10 a.m.?
- 2. To improve her physical condition, Arianna swims, bikes, and runs.
   Every day Arianna swims 40 lengths of a pool that is 25 meters long. How far does Arianna swim each day?

**3.** Marites has read  $\frac{1}{3}$  of a 240-page book. How many pages has she  $\binom{lnv.3}{46}$  read? What percent of the book has she read?

**4.** If 3 tickets cost \$12, how many tickets can Cole buy with \$20?

**\*5.** Arrange these fractions in order from least to greatest:

5	3	2	1
5'	4'	6'	2

\*6. Analyze A number is divisible by 4 if it can be divided by 4 without leaving a remainder. The numbers 8, 20, and 32 are all divisible by 4. What number between 10 and 20 is divisible by both 4 and 6?

**7.** Use a fraction and a decimal number to name the shaded portion of this square:

\*8. Which digit in 16.43 is in the tenths place?

\*9. **Represent** The length of the notebook paper was 0.279 meter. Write 0.279 with words.

\*10. Connect Use a mixed number and a decimal number to name the point on this number line marked by the arrow:



**\* 11. Represent** Write the decimal number 0.03 as a fraction.

- \* **12.** A jewelry designer used 81 grams of gold alloy to make 10 identical earrings. What was the weight in grams of the gold alloy in each earring?
  - **13.** The length of  $\overline{RT}$  is 100 millimeters. If the length of  $\overline{RS}$  is <sup>(61)</sup> 30 millimeters, then how long is  $\overline{ST}$ ?

<b>14.</b> 87,906 71,425 + 57,342	<b>15.</b> 407 ( <sup>(56)</sup> × 819	<b>16.</b> \$8.76 (26) \$6
<b>17.</b> 600 ÷ (60 ÷ 6)	<b>18.</b> 40)5860	

**19.** If each side of a regular hexagon is 4 inches long, then what is the perimeter of the hexagon?

**20.** 341 + 5716 + 98 + 492 + 1375

**21.**  $7 \times 6 \times 5 \times 4$  **\*22.**  $5\frac{1}{4} - \left(3 - 1\frac{3}{4}\right)$  **\*23.**  $3\frac{1}{6} + 2\frac{2}{6} + 1\frac{3}{6}$ **24.** 20w = 300

```
25. Compare: 365 × 1 ○ 365 ÷ 1
```

**26.** (46) William's company made \$30,000 last month. As the owner, William received one tenth of the money. How much money did William receive? Explain how you found your answer.

\* 27. **Conclude** In this figure there are three triangles. Triangle *WYZ* is a right triangle. Which triangle appears to be an obtuse triangle?



**28.** A coin is tossed once.

- a. List all the possible outcomes.
- b. What fraction describes the probability of each outcome?
- \*29. **Represent** Write 0.625 with words.
- \*30. **Represent** Use digits to write the decimal number twelve and seventy-five hundredths.



Power Up

## Comparing and Ordering Decimal Numbers

facts	Power Up F				
equivalent fractions	The following fractions are equal to one half: $\frac{1}{2}$ , $\frac{2}{4}$ , $\frac{3}{6}$ . Read the fractions aloud and continue the pattern to $\frac{12}{24}$ .				
mental math	a. Percent: 10% of \$100				
	<b>b. Percent:</b> 10% of \$10				
	c. Powers/Roots: $\sqrt{100}$				
	<b>d. Number Sense:</b> $3\frac{1}{2} + 3\frac{1}{2}$				
	e. Probability: Ava wrote the name of each month on separate cards. She then turned the cards over and mixed them up. If she picks up one card, what is the probability it will be the card labeled "May"?				
	f. Time: Six years is how many months?				
	<b>g. Fractional Parts:</b> The sale price is $\frac{1}{3}$ off the regular price. What is the discount on a desk that is regularly priced at \$120?				
	<b>h. Calculation:</b> $\frac{1}{3}$ of 12, $\times$ 5, $-2$ , $\div$ 2, $\times$ 5, $-1$ , $\div$ 4				
problem solving	Choose an appropriate problem-solving strategy to solve this problem. All squares are similar. Each side of this 1 cm square is 1 centimeter long. Draw a square with sides twice as long. Calculate the perimeter of each square. Then estimate how many of the smaller squares could fit inside the square you drew. Explain how you arrived at your answer.				



Fractions of a second are usually expressed as decimals.

Cecilia ran 100 meters in 14.6 seconds. Marlon swam 50 meters in 28.43 seconds.

Cecilia's 100-meter time was fourteen and six tenths seconds. However, athletes often state their race times in a shorter way. Cecilia might say she ran "fourteen point six" or even "fourteen six." If she runs 100 meters in 14.0 seconds, she might say she ran "fourteen flat." What is important to understand is that 14.6 seconds is more than 14 seconds but less than 15 seconds. A tenth of a second is a short period of time; it is about how long it takes to blink your eyes. A hundredth of a second is even shorter. Races timed to a hundredth of a second are timed electronically rather than by a hand-held stopwatch because a person with a stopwatch cannot react quickly enough to get an accurate reading.

# Fractions of a Second

Material needed:

stopwatch

A stopwatch can help us understand fractions of a second. If a stopwatch is available, try these activities:

- Test your ability to estimate brief periods of time. Without looking at the stopwatch display, start the watch and then try to stop it at 5 seconds. Record the time shown on the watch. Repeat the experiment once, and then calculate how close each estimate was to 5 seconds. Which estimate was closer? How close to 5 seconds did you get?
- **Test your speed.** Start and then stop the stopwatch as quickly as you can. Repeat the experiment once and record the shorter of the two times.

To compare decimal numbers, we need to pay close attention to place value. The decimal point separates the whole-number part of a decimal number from the fraction part.

#### Thinking Skill

#### Verify

How many seconds are equal to...

one minute? one hour? one day?

#### Example 1

#### Compare: 12.3 () 1.23

Although the same digits appear in both numbers in the same order, the numbers are not equal. The number 12.3 is a little more than 12, but it is less than 13. The number 1.23 is more than 1 but less than 2. So 12.3 is greater than 1.23.

#### 12.3 > 1.23

Example 2

#### Arrange these numbers in order from least to greatest:

#### 1.02, 1.2, 1.12

Arranging the numbers vertically with the decimal points aligned can make the order easier to determine. We compare the digits column by column, beginning with the first column on the left.

1	.02
1	.2

1.12

The whole-number part of each number is 1, so we need to compare the fraction parts. The first digit to the right of the decimal point is in the tenths place. (In money, it is the dimes place.) The number 1.02 has a zero in the tenths place, the number 1.12 has a one in the tenths place, and the number 1.2 has a two in the tenths place. This is enough information to order the numbers from least to greatest.

#### 1.02, 1.12, 1.2

- **a.** Alec ran 200 meters in 38.6 seconds. Carina ran 200 meters in 37.9 seconds. Which athlete ran faster?
- **b.** Compare: 3.21 () 32.1
- c. Arrange these numbers in order from least to greatest:

2.4, 2.04, 2.21

Written Practice

Distributed and Integrated

- 1. The ceiling was covered with square tiles. There were 30 rows of tiles
- with 30 tiles in each row. How many tiles covered the ceiling?

Thinking Skill

#### Connect

What would these amounts be in dollars and cents?

Lesson Practice

- **2.** Sunee gave the clerk \$10 for a book that cost \$6.95 plus \$0.42 tax. How much change should she receive?
- **3.** Matia emptied a jar of 1000 pennies and put them into rolls that held <sup>(21)</sup> 50 pennies each. How many rolls did she fill?
- **4.** The distance around the school track is  $\frac{1}{4}$  mile. How many times must Brandon run around the track to run 1 mile?
  - **5. Analyze** What even number greater than 20 and less than 30 is divisible by 3?
  - 6. List Write the factors of 10 that are also factors of 15.
- \*7. Compare: 44.4 () 4.44
- 8. Which digit in 56,132 is in the same place as the 8 in 489,700?
- \*9. Use both a fraction and a decimal number to name the <sup>(67)</sup> unshaded portion of this group of circles.



\* **10.** Find the length of this segment to the nearest tenth of a centimeter.

	Π	 IIII	T	ηп		
cm	1		2		3	4

\* **11.** Which digit in 67.89 is in the hundredths place?

**12.** The length of  $\overline{LN}$  is 4 inches. If  $\overline{MN}$  is  $1\frac{1}{2}$  inches, then how long is  $\overline{LM}$ ?

- \* **13. Represent** Write 10.5 with words.
- \* **14. Represent** Use digits to write the decimal number fifteen and twelve hundredths.

15.	<u>3744</u>	<b>16.</b> 30,000 <b>17.</b>	973
(26)	0	(9) - 29,925	× 536

**18.** What number is half of 75?

**19.**
 \$0.65
 **20.**

$$5$$
)
  $5$ )
 **21.**
 \$54.30

 (29)
  $\times$ 
 10
 (26)
  $5$ )
  $5$ )
  $30$ 
**\* 22.**
 $7 - \left(3 + 1\frac{1}{3}\right)$ 
**\* 23.**
 $5\frac{2}{3} + \left(3\frac{1}{3} - 2\right)$ 
 $30$ 

\* 24. Use this information to answer parts **a** and **b**: (11, 35) In the school election for president, Aaron received 239 votes, Brigit received 168 votes, and Phuong received 197 votes.

- **a.** One other person ran for president and received 95 votes. Altogether, how many votes were cast for president?
- **b.** The winner received how many more votes than the person who came in second?
- **\*25.** A number cube is rolled once. What is the probability of each of these outcomes?
  - **a.** The number will be 6 or less.
  - **b.** The number will be greater than 6.
  - c. The number will be even.
- \*26. What is the place value of the 7 in \$6.75?

**\*27. Represent** Name the shaded portion of this square as a fraction, as a decimal number, and with words.




\*29. Interpret The line graph shows temperatures from 5 a.m. to 11 a.m.

(Inv. 5, Inv. 6) one winter morning in Grand Forks, North Dakota. Use the graph to answer the questions that follow.



- **a.** What number of degrees represents the range of the temperatures from 5 a.m. to 11 a.m.?
- **b.** During which two-hour period of time did the greatest temperature increase occur? What was that increase?
- **c. Explain** How many degrees below the freezing temperature of water was the 5 a.m. temperature? Explain your answer.
- **30.** During summer vacation, Khara visited the skateboard park 3 more times than Brooke, and Brooke visited 7 fewer times than Tamika. Brooke visited the park 15 times. How many times did Khara and Tamika each visit the park?



Many libraries use a system of decimals called the Dewey Decimal System to classify books and order them on the shelves. Three math books are numbered 510.865, 510.866, and 510.86. Order these numbers from least to greatest.



# Writing Equivalent Decimal Numbers

Power Up

facts	Power Up G
mental math	A number is divisible by 4 if the number formed by the last two digits is a multiple of 4. For example, 1324 is divisible by 4 because 24 is divisible by 4, but 1342 is not. Use this information to answer problems <b>a-d</b> .
	a. Number Sense: Is 1234 divisible by 4?
	<b>b. Number Sense:</b> Is 3412 divisible by 4?
	c. Number Sense: Is 2314 divisible by 4?
	d. Number Sense: Is 4132 divisible by 4?
	<b>e. Number Sense:</b> 100 ÷ 4
	<b>f. Number Sense:</b> 200 ÷ 4
	<b>g. Number Sense:</b> 300 ÷ 4
	<b>h. Calculation:</b> $\frac{1}{4}$ of 36, + 1, × 2, ÷ 4, × 3, - 1, ÷ 7
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Amol, Badu, Conrad, and Delores were posing for a picture, but the photographer insisted that only three people could pose at one time. List the combinations of three people that are possible. (In this problem, different arrangements of the same three people are not considered different combinations.)
New Concept	
	We may attach one or more zeros to the end of a decimal

We may attach one or more zeros to the end of a decimal number without changing the number's value. For example, we may write 0.3 as 0.30. The zero does not change the value of the number because it does not change the place value of the 3. In both numbers, 3 is in the tenths place. Thus, three tenths is equal to thirty hundredths.

## Example 1

Write 12.6 with three decimal places.

The number 12.6 is written with one decimal place. By attaching two zeros, we get **12.600**, which has three decimal places.

**Connect** How would you read 12.600?

#### **Example 2**

#### Compare: 12.6 () 12.600

When we compare decimal numbers, we must pay close attention to place value. We use the decimal point to locate places. We see that the whole-number parts of these two numbers are the same. The fraction parts look different, but both numbers have a 6 in the tenths place. If we add two zeros to 12.6 to get 12.600, we see that the numbers are the same, so we use an equal sign in the comparison.

#### 12.6 = 12.600

Discuss Why is 6 tenths equal to 600 thousandths?

Here are two ways to write "fifty cents":

- 1. As a number of cents: 50¢
- 2. As a number of dollars: \$0.50

Sometimes we see signs with a money amount written incorrectly.



This sign literally means that a can of juice costs  $\frac{50}{100}$  of a penny, which is half a cent! The sign could be corrected by changing 0.50¢ to \$0.50 or to 50¢.

# Use digits and symbols to write "five cents" both in cent form and in dollar form.

The cent form is **5¢.** The dollar form is **\$0.05.** 

Example 3

Example	4	•••••	• • • • • • • • • • • • • • • • • • • •	······
	This sign is written ind two ways to correct th amount shown on the	correctly. S ne money sign.	Show	Bananas <sup>0.25¢</sup>
	We may write <b>25¢</b> (cent (dollar form).	t form) or <b>\$</b>	0.25	
2	••••••	•••••	••••••	
Example	5	• • • • • • • • • • • • • • • • • • • •	• • • • • • • • • • • • • • • • • • • •	••••••
	Reuben bought a pace \$1.56 and a folder for did he spend?	kage of pa 75¢. How i	per for much	\$1.56 + \$0.75 \$2.31
	When both forms of mo problem, we first rewrite that they are all in the s solve the problem. Sum to a dollar or more are u \$1.56 + 75¢, we can ch	oney are in e the amou ame form. ns of money usually writ nange 75¢	the same nts so Then we y equal ten with a dol to dollar form	lar sign. To find and then add. as
	shown above. Jorge sp	ent <b>\$2.31.</b>		
Lesson Practice	Write each number w	vith three de	ecimal places	
	<b>a.</b> 1.2	<b>b.</b> 4.08	C. (	0.50000
	Compare:			
	<b>d.</b> 50 🔵 500		<b>e.</b> 0.4 () 0.	04
	<b>f.</b> 0.50 🔵 0.500		<b>g.</b> 0.2 🔿 0.	20000
	<b>Represent</b> Write ea dollar form:	ach money	amount in ce	nt form and in
	h. two cents		i. fifty cents	i
	j. twenty-five cents	6	k. nine cents	6
	Solve problems I-o.	Write each	answer in the	e indicated form.
	<b>I.</b> 36¢ + 24¢ = \$		<b>m.</b> \$1.38 - 7	∕0¢ =¢
	<b>n.</b> \$0.25 − 5¢ = \$_		<b>o.</b> \$1 - 8¢ =	=¢
	Multiply. Write each p	product in c	lollar form.	
	<b>p.</b> 7 × 65¢		<b>q.</b> $20  imes 18$ ¢	

### Written Practice

Distributed and Integrated

- **1. Analyze** Each side of a 1-foot square is 1 foot long. What is the perimeter of a 1-foot square?
- Anna Pavlova was a world-famous Russian ballerina born in 1881.
   <sup>(35)</sup> George Balanchine, one of the founders of the New York City ballet, was born in 1904. How many years before the birth of George Balanchine was Anna Pavlova born?
- **3. Estimate** Find the product of 307 and 593 by rounding both numbers to the nearest hundred before multiplying.
- **4. Evaluate** Three times a number *n* can be written "3n." If *n* equals the number 5, then what number does 3n equal?
- **\*5.** Tyrique calculates that a weekend is 48 hours long and that he sleeps  $\binom{lnv.3}{60}$  for 16 hours each weekend, or  $\frac{1}{3}$  of the weekend. What fraction of each weekend does Tyrique spend awake? What percent represents that fraction?

**\*6. Represent** Draw a circle and shade one eighth of it. What percent of (*lnv. 3, 37*) the circle is shaded?

- **\*7.** (58) Can 100 students arrange themselves in 7 different teams if there are to be the same number of students on each team? Explain why or why not.
- **\*8.** Which digit in 12.3 is in the tenths place?
- \*9. **Connect** Use a fraction and a decimal number to name the shaded part of this square:



\* **10.** Which digit in 98.765 is in the thousandths place?



**b.** with a cent sign.

**24.** The thermometer shows the hottest temperature ever recorded on the continent of Australia. What was that temperature?



\*25. Suppose the 8 letter tiles below are turned over and mixed up. Then one tile is selected.



Which word best describes the following events: likely, unlikely, certain, or impossible?

- a. The letter selected is a consonant.
- **b.** The letter selected comes after S in the alphabet.
- **c.** The letter selected is either G or H.
- **26.** Courtney and Lamar went fishing for trout. They caught 17 trout that were at least 7 inches long. The distribution of lengths is shown on the line plot below. Refer to this information to answer parts **a-c.**



- a. How many trout were less than 11 inches long?
- **b.** Which lengths were recorded more than three times?
- **c.** Which of the lengths, if any, are outliers?

**27.** Refer to the line plot in problem **26** to answer parts **a–c**. (Inv. 5

- **a.** What is the median?
- **b.** What is the mode?
- **c.** What is the range?
- **\*28. Connect** One fourth of this square is shaded. Write the (67.68) shaded portion of the square as a decimal number. Then write the decimal number with words.

**29.** (49) The star Fomalhaut is about 25 light years from Earth. The star Gacrux is about 63 light years farther from Earth than Fomalhaut, and the star Hadar is about 437 light years farther from Earth than Gacrux. About how many light years distant from Earth is the star Hadar? Explain how you found your answer.

\*30. (62) **Estimate** One way to estimate the product of  $76 \times 4$  is to round round 76 to 80 and find the product of  $80 \times 4$ . Describe another way to estimate the product of  $76 \times 4$ .

Early Einishens Real-World

Connection

Several students in Jenna's science class planted beans in soil. Thestudents kept a growth chart for the plants. The chart below shows the height of the plants after two weeks.

Student	Height of Plant
Jenna	2.5 cm
Juan	2.05 cm
Lydia	2.55 cm
Peyton	2.50 cm

- **a.** Name the two students whose plants reached the same height. Explain how you know.
- **b.** Arrange the plant heights in order from least to greatest.

### Focus on

## Displaying Data

Data that are gathered and organized may be displayed in various types of charts and graphs. One type of graph is a **bar graph**. A bar graph uses rectangles, or bars, to display data. Below we show the test scores and frequency table from Investigation 5 and a bar graph that displays the data.

Test scores: 4, 3, 3, 4, 2, 5, 6, 1, 3, 4, 5, 2, 2, 6, 3, 3, 4, 3, 2, 4, 5, 3, 5, 5, 6



Notice how the information from the frequency table is presented in the bar graph. The scale across the bottom of the bar graph (the horizontal axis) lists all the possible scores on the tests. It shows the same values as the first column of the frequency table. The scale along the left side of the bar graph (the vertical axis) lists the number of students. The height of a bar shows how often the score shown below the bar was achieved. In other words, it shows the frequency of the score.

Now we will practice making bar graphs using a new situation. Twenty children in a class were asked how many siblings (brothers and sisters) they had. The data from their responses, as well as a frequency table to organize the data, are shown below.

> Number of siblings: 2, 3, 0, 1, 1, 3, 0, 4, 1, 2, 0, 1, 1, 2, 2, 3, 0, 2, 1, 1

**Frequency Table** 

Number of Siblings	Tally	Frequency
0		4
1	JHI II	7
2	Ш	5
3		3
4		1

**1. Represent** Copy and complete this bar graph to display the data from the frequency table:



In Investigation 5, we made frequency tables with data grouped in intervals of equal size. From that investigation, recall that ABC Market offered turkeys with these weights (in pounds):

11, 18, 21, 23, 16, 20, 22, 14, 16, 20, 17, 19, 13, 14, 22, 19, 22, 18, 20, 12, 25, 23

Below is the frequency table for these data using intervals of 4 pounds, starting with the interval 10–13 pounds.

ricquonoy fubio		
Weight (in pounds)	Tally	Frequency
10–13		3
14–17	Ш	5
18–21	JH III	8
22–25	JHI I	6

#### **Frequency Table**

To graph data grouped in intervals, we can make a **histogram**. A histogram is a type of bar graph. In a histogram, the widths of the bars represent the selected intervals and there are no spaces between the bars. Below is a histogram for the turkey-weight data. The intervals in the histogram match the intervals in the frequency table.



2. **Represent** Create a frequency table and a histogram for the turkey weights using these intervals in pounds:

11-13, 14-16, 17-19, 20-22, 23-25

Another way to display these turkey weights is in a **stem-and-leaf plot.** The "stems" are the tens digits of the weights. The "leaves" for each stem are the ones digits of the weights that begin with that tens digit. The stem-and-leaf plot for the first row of weights in the list is shown below. Notice that the leaves are listed in increasing order.

- **3. Represent** Make a stem-and-leaf plot for the second row of weights in the list.
- **4.** Use the information in the stem-and-leaf plots for the first and second rows of weights to make a stem-and-leaf plot for the weights of all 22 turkeys.

Numerical data represent quantities such as ages, heights, weights, temperatures, and points scored. Data can also come in **categories**, or **classes.** People, concepts, and objects belong to categories. Examples of categories include occupations, days of the week, after-school activities, foods, and colors.

Suppose Amelia asked the students in her class to name their favorite type of juice and then displayed the data in this frequency table and bar graph:



The bar graph Amelia made is called a *horizontal bar graph* because the bars run horizontally. The categories that Amelia used for her data are types of juice: grape, cranberry, apple, and orange.

Sixty students were asked to give their position among the children in their family. Their responses were put into the four categories shown in this frequency table:

Frequency Table		
Category	Frequency	
Only child	22	
Youngest child	16	
Oldest child	14	
Middle child	8	

5. **Represent** Make a horizontal bar graph for the data in the table above. Be sure to label each bar along the vertical side of the graph with one of the four categories. Along the bottom of the graph, use even numbers to label the number of students.

Recall that a **pictograph** uses symbols, or **icons**, to compare data that come from categories. An icon can represent one data point or a group of data points. In pictographs we include a **legend** to show what the icon represents.

Suppose 96 children were asked to choose their favorite sandwich from grilled cheese, tuna fish, and smoked turkey. The data that was collected is displayed in the pictograph on the next page.

	Favorite Sandwich
Grilled cheese	
Tuna fish	
Smoked turkey	$\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$

Key: = 4 students

**Interpret** We count 8 symbols for tuna fish in the pictograph. To find how many children 8 symbols represent, we read the legend and find that we should multiply the number of symbols by 4. We multiply  $8 \times 4$  to find that 32 children prefer tuna fish.

- 6. How many children prefer grilled cheese?
- 7. How many children prefer smoked turkey?
- **8.** Draw a second pictograph for the food preferences in which each symbol represents 8 children.

Sometimes we are interested in seeing how categories break down into parts of a whole group. The best kind of graph for this is a **circle graph**. A circle graph is sometimes called a *pie chart*.

**Interpret** The following circle graph shows how Greg spends a typical 24-hour day during the summer:



- **9.** On which two activities does Greg spend most of his time? Which activity does Greg spend the least amount of time on?
- 10. Which activity takes about the same amount of time as sports?
- 11. Which activities take more time than sports?
- 12. List activities that together take about 12 hours.

In the examples we have presented, the categories do not overlap. This means that each data point fits in only one category. We say that such categories are **mutually exclusive**. But sometimes we have data points that fit into more than one category. To display such data, we can use a **Venn diagram**. The categories in a Venn diagram are represented by overlapping circles.

Suppose Karim surveys his 21 classmates about family pets. Karim asks whether they have a dog at home, and he asks whether they have a cat at home. There are four possibilities. A family could have both kinds of pets. A family could have a dog only. A family could have a cat only. Finally, a family could have neither kind of pet.

Karim finds that 12 of his classmates have dogs and 8 have cats. Of these 12 students, 5 have both. We will make a Venn diagram to display this information.

We begin by drawing two overlapping circles. One circle represents the families with dogs. The other circle represents the families with cats. We must make the circles overlap because some families have both dogs and cats. We are told that 5 families have both kinds of pets, so we write "5" in the overlapping region.

We have accounted for 5 of the 12 families who have dogs, so there are 7 families that have dogs but that do not have cats. We write "7" in the region of the diagram for families with dogs only.

We have also accounted for 5 of the 8 families that have cats, so there are 3 more families that have cats. We write "3" in the region of the diagram for families with cats only.

We have now accounted for 7 families with dogs only, 5 families with both dogs and cats, and 3 families with cats only. That is a total of 15 families. However, 21 families were part of the survey, so 6 families have neither animal. We can write "6" outside the circles to represent these families.

**13. Analyze** How many families had just one of the two types of animals?

Suppose Tito asked each of his classmates whether he or she plays soccer.

**Analyze** Refer to this Venn diagram he created to answer problems **14–19**.



- 14. How many boys are in Tito's class?
- **15.** How many soccer players are in Tito's class?



6

- 16. How many of the boys play soccer?
- 17. How many soccer players are girls?
- 18. If there are 30 students in his class, how many girls are there?
- **19.** If there are 30 students in his class, how many girls do not play soccer?



- **a.** In which month were you born? Create a horizontal bar graph that displays the birth months of your classmates.
- b. In which season were you born: winter, spring, summer, or fall? (Use December 22, March 22, June 22, and September 22 as the first day of each season.) Create a histogram that displays the seasons in which your classmates were born.
- **c.** Survey each student in your class, and record the answers to these two questions:
  - Do you like to watch cartoons on television?
  - Do you like to watch sports on television?

Copy and complete the Venn diagram below to display the number of students who answer "yes" to either or both questions.



- **d.** Track local high temperatures for a week, and make a line graph of the daily high temperatures. Write two problems involving temperature changes that could be solved using the line graph you made.
- e. Survey ten friends about a favorite book or movie, and make a pictograph of the data. Write two problems that could be solved using the pictograph you made.
- f. Select four classroom items that weigh between 100 grams and 1 kilogram. Using a scale or balance, determine the items' actual weights. Choose a type of graph to best display the data you have collected. After creating the graph, explain why you chose that particular type of graph.